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Natural Disaster Risk Bearing Ability of Governments: Consequences of kinked utility

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ABSTRACT

In this paper the risk neutrality paradigm for government stocks and investments is challenged within the context of catastrophe risk. We focus on government's ability to spread its natural disaster risk. Based on the classical approach of Arrow and Lind, the paper shows the weaknesses of and reformulates the risk neutral assumption for government decisions under uncertainty. The rationale that governments have kinked utility functions, which can arise from natural disaster events, is given through a network example and its implications explained considering also risk aversion and the benefits of different types of risk management strategies.

Keyword: Catastrophe risk, kinked utility, government, risk bearing ability, natural disasters, networks.

1. Introduction

The magnitude of economic and insured losses from natural disaster events seem to exhibit a rising trend (Munich Re, 2007). As a consequence proactive risk management strategies are becoming increasingly important for reducing potential disaster losses and lessening their long term impacts (Gurenko, 2004). Different stakeholders can or may use different sets of options for managing their risk. On the country level, risk bearers can be grouped into the government, the domestic private sector and international institutions (Miller and Keipi, 2005). The private sector can be further distinguished as property owners (households and businesses), insurers, reinsurers and the capital market (Grossi and Kunreuther, 2005). The risk management strategies available to them include risk reduction, risk transfer and risk spreading mechanisms. However, their risk management strategies are often a function of their perception of the risk they are exposed to (Loefstedt and Frewer, 1998; Slovic, 2000). Low probability but high consequence events are especially neglected for a number of reasons. For example, people may set (subjective) threshold levels for probability such that an event below the threshold is not considered possible. As a consequence, this event is not incorporated in their decision making process (Kunreuther, 1996). If the risk is however realized, the high losses can result in the inability to (efficiently) cope with the event and may have serious negative consequences in the future, e.g. high indebtedness over a long time period which limits development options. This situation is not restricted to the household level or business sector, but can also apply for the public sector (Hochrainer, 2006).

The ability to deal with risks can be described as risk-preference. This human characteristic can be modelled through the use of *utility functions*. While there are now a variety of other approaches to analyze decisions under risk, e.g. prospect theory (see Kahneman and Tversky, 1979), in this paper we use the classical approach to show and reformulate from a theoretical as well as practical standpoint some of the weaknesses of the risk neutral assumption for government decisions under uncertainty. Hence, our main focus is on the government and its ability to spread its risk from natural disaster events. From a theoretical point of view, one must consider the Arrow-Lind theorem (1970), which states that governments should behave risk-neutrally and therefore evaluate their investments and current stock under uncertainty only through the expected present (social) value (Little and Mirrless, 1974). The paper discusses and reformulates its weaknesses in practical situations motivated by concrete examples and gives a proof for a special case of utility functions, namely with kinks. That such kinked functions appear also in practice is shown through a network example. Furthermore, the problem of contingent liabilities is discussed within this example for showing the marginal benefits of different risk management strategies. While the treatment of kinked utility functions for decision makers was already analyzed by others (see Segal and Spivak, 1990), we restate and reformulate it from a government perspective.

The paper is organized in two parts: a theoretical section which discusses the Arrow-Lind theorem and related topics, and a more practical part where a simple network example should underline the arguments made before, and implications of these are discussed. In detail, section 2 starts with a reformulation of the Arrow-Lind theorem and gives a short literature review. Section 3 summarizes the weaknesses of the assumptions from a practical perspective, and in section 4 a network example is given to motivate the argument that kinked utility functions may also be present for governments. In section 5 a formal proof of why governments should behave risk aversely in such situations is given. Section 6 then discusses possible risk management strategies the government may use because of its inability to spread risk and discusses the benefits of mitigation and loss financing strategies.

2. Arrow-Lind Theorem: Discussion

Kenneth J. Arrow and Robert C. Lind investigated the question of how governments should treat uncertainty in the evaluation of public investment decisions (Arrow and Lind, 1970). They argue that if the risks associated with a public investment are publicly borne (e.g. through taxation), the total cost of risk-bearing is insignificant, and therefore governments should ignore uncertainty. The reason behind this is that governments can efficiently transfer risk through their ability to distribute losses over such a large population that the per capita loss for each individual is negligible. Besides this risk spreading ability, governments are also able to pool a large number of risks. The pool ideally consists of a large number of independent public infrastructure assets in various regions of the country. Thus the government's risk portfolio is diversified and risk, therefore, can be neglected. Consequently, governments should behave risk-neutrally and evaluate their investments only through the expected net present (social) value (Little and Mirrless, 1974).

In more detail, when the government undertakes an investment, each taxpayer has a small share of that investment with the returns being paid through changes in tax levels. Assuming risk aversion for taxpayers, the value of the asset to the individual is less than its expected return. Hence, to compute the value of the investment to the individual taxpayer, a risk-bearing cost (factor) must be subtracted from the expected return. To prove that the total of the costs of risk-bearing goes to zero as the population of taxpayers becomes large, Arrow and Lind assumed a utility function U which is bounded, continuous, strictly increasing and differentiable. Furthermore, all individuals are identical, have the same preferences, their incomes are identically distributed variables and they are subjected to the same tax rates. The assumptions about the taxpayers were made to simplify the proof, but they are not essential to the argument. Their proposition can be stated in mathematical terms as:

Proposition 1. *If* U *is a twice differentiable and strictly monotonic utility function, satisfying* $|U(u)|\leq$

$K(1 + u^2)$ and X is a random variable with $\mathbb{E}[X]=0$, $\mathbb{E}[X^2] < \infty$*, then for all a*

$$\lim_{n\to\infty} n[a - U^{-1}(\mathbb{E}[U(a + X/n)])] = 0 \quad (1)$$

The quantity

$$\pi_n = [a - U^{-1}(\mathbb{E}[U(a + X/n)]$$

is called the individual risk premium.

The technical assumption that U is bounded by some other function, e.g. $|U(u)| \leq K(1 + u^2)$, is needed for applying the dominated convergence theorem, i.e. to interchange limit and integral operations within the proof. π_n is introduced using the certainty equivalent, i.e. $\mathbb{E}[U(a + X/n)] = U(a + \pi_n)$.
The Arrow-Lind proposition supports the argument that governments should behave risk-neutrally, because dividing risk X by n individuals drives the collective risk premium $n\pi_n$ (i.e. the sum of all individual risk premiums) to zero as n increases. Summarizing, the collective risk premium $n\pi_n$ is close to zero, if (i) n is large, (ii) $\mathrm{Var}(X)$ is small and (iii) $U''(a)$ is small. Section 3 will reformulate these mathematical reasons in more qualitative ones.

A lively and fruitful discussion was started (and continues presently) after Arrow and Lind published their paper. Criticisms on the assumptions made by Arrow and Lind were raised (Mishan, 1972; McKean and Moore, 1972; Nichols, 1972; Wellington, 1972). The Arrow Lind setup was further investigated in the case of environmental externalities where public investments produce uncertain environmental side effects which must be taken into account in evaluating public investment decisions (Fisher, 1973). Further analyses, which made the roles of the various assumptions more transparent in order to bring out the implications of a more realistic specification of the fiscal system in which public sector investments are embedded, were performed by Foldes and Rees (1977). They concluded that the conditions set by Arrow and Lind must be more stringent than originally defined, and that the circumstances in which the conclusions of the theorem apply are very restricted. Gardner (1979) investigated the Arrow-Lind theorem in a continuum economy were the taxpayers belong to the set $T = [0, 1]$ were a individual agent is an infinitesimal subset dt of T. He concluded that only for small government risks is the total cost of risk bearing necessarily insignificant. The main result, being the ability of the government to efficiently spread risk, is still controversial today. For example Blake and Burrows (2001) used the risk spreading argument to conclude that in the context of mortality risk, the government should issue survivor bonds. Contrary to this, Dowd (2003) argued that capital markets are better suited than any government to bear and share risks, because this is their original purpose (see for example Priest (2003) for a general discussion about public and private insurance in the context of the Arrow-Lind theorem).

In this paper, a closer look at the ability of the government to spread risk in the special case of catastrophic events is made. Catastrophic events form a class of their own and have therefore to be analysed separately. In the next section, some of the problems in this context are presented from a practical point of view, to further motivate the issues.

3. Weaknesses and Limitations in Practice

First, some examples of when the theorem seems to work and when not are provided. Arrow and Lind argue that the risk-bearing cost will be small if the size of the share borne by each taxpayer is negligible in comparison to his/her income. It appears reasonable that this indicates also that the total cost of risk bearing is small. This is because under such circumstances the investment is also small with respect to the total wealth of the taxpayers. In the case of a federally sponsored investment, the number of taxpayers is not only large, but also the investment represents only a small fraction of the national income and therefore the risk-bearing costs are small even when the investment itself appears large in absolute terms. Such situations seem to be the case in developed countries with wealthy and stable economies. For example, in Germany the losses resulting from the 2002 flooding were dramatically high, with direct losses of approximately 9.2 billion Euro (Munich Re, 2003). But mainly from taxation (Sonderfonds Aufbauhilfe), more money for financing than needed was available (Werner et al., 2003). However, in developing countries one often finds a different picture. For example, Hurricane Mitch caused direct damages in Honduras equivalent to 2 billion dollars (Mechler, 2004a). This is much lower than the 2002 flood losses in Germany, but recognizing that more than 50 percent of the 6.2 million people in Honduras live below the poverty line, a roughly estimated cost of 320 US$ per person if the losses are spread over the entire population indicates that additional taxation would not be feasible. Hence, with a limited number of taxpayers the cost to individuals depends upon the absolute size of the total cost to be spread and the actual number and wealth of the taxpayers. Additionally, as developing countries usually undertake just a few large projects such that risk pooling is not an appropriate consideration (Brent, 1998), the situation is for them even more problematic. Hence, the theorem can be quite misleading in practice if it is taken for granted in all situations.

Let us now return to proposition 1. We already stated that the collective risk premium approaches zero only under some certain conditions. Stated differently, the proposition is not applicable, if

- countries have relatively small populations (n cannot increase very much)
- the risk is relatively large (e.g. the variance of X is large),
- even small per capita risk is not negligible,
- U is very nonlinear or nondifferentiable.

The first and second points are usually fulfilled by small island states and most developing countries. Contrary to common health or injury risks, natural catastrophe risks are geographically correlated. This means that for small countries there is a higher probability of larger losses relative to population. For developing countries with less developed early warning systems or adherence to construction codes, relative losses will be even larger. Furthermore, in developing countries with widespread poverty, even small damages or small income shocks may not be negligible. Concerning the third point, when even small per capita risk is not negligible, the individual's utility function could be interpreted as high curved, implying that the Arrow-Pratt risk aversion coefficient (Pratt, 1964) is high. The reason for this is that the wealth of these individuals is very low, so that also small capita risk are seen as not negligible. Mechler (2004b) summarized and also reformulated the qualitative conditions when the Arrow-Lind theorem should not or can not be used to assume risk neutrality for the government:

- Countries subject to high natural hazard exposure
- Countries subject to high economic vulnerability
- Countries with few large infrastructural assets and high geographical correlation between those assets
- Countries with concentrated economic activity centers exposed to natural hazards.

Additional to these points above, there is also another important reason why the risk neutral argument does not apply in the case of natural catastrophes, having mainly to do with the assumptions of the utility function. The implications are interesting from a theoretical point of view and also have practical implications. We start with a simple network example for showing that kinked utility functions are quite natural and need to be considered in catastrophic events, also from a government's perspective.

4. Non-optimal destruction of Networks

Rationales for using utility functions with a kink can be found in behavioral frameworks such as prospect theory to distinguish people's different preferences with respect to gains and with respect to losses relative to a reference point (Kahnemann and Tver-sky, 1979,1992; Starmer 2000) or similarly in first-order risk aversion models (with kinked utility) where losses more heavily weighted than gains in the investor's utility (Epstein and Stanley, 1990; Gul, 1991) as well as in insurance type of analysis, e.g. to determine the appropriateness of unregulated markets for human wealth and liability risks (Sinn, 1982). In this section it is illustrated how kinked utility functions can arise in a natural way for catastrophe risk. While utility functions are typically considered as artefacts, which are unobservable and can only be indirectly guessed by observing preference decisions, if available, concrete benefit functions as they appear in various OR applications are preferable as utility functions.

In the following, a network example is constructed, as for instance a road system, for showing how kinked utility functions may appear. Here, the costs are the investments in the arcs (the roads), while the utility is the total throughput through the system. For simplicity, we consider a directed network and just one origin-destination throughput. The difference between network construction by planned investment and network destruction by natural catastrophes becomes evident in this example: While infrastructure construction is typically based on an optimal cost-benefit decision, the infrastructure destruction by natural catastrophes occurs in an erratic manner. This leads in a natural way to different marginal utility increases for construction and decreases for destruction. To be more concrete, consider the following directed network (Figure 1):

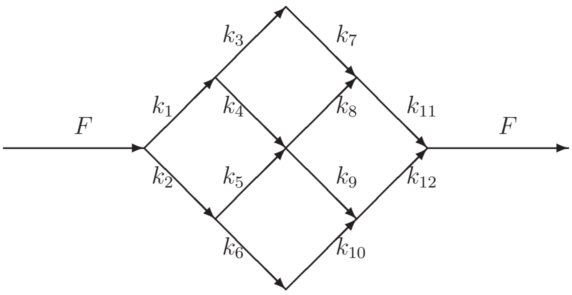

Figure 1. A network example: The flow F represents the return. Investments can be made to increase the arc capacities k_i.

The government as investor may invest in the capacities of the arcs. The capacity k_i of arc i is a function $k_i(c_i)$ of the amount c_i invested in this arc. The utility of the investment is the maximal flow F through this network. For a given budget B, the optimal investment strategy maximizes total network flow under the budget constraint. Denote this maximal flow by $U(B)$. Figure 4 shows this function as a dotted line. The concrete specifications of this example are given in Appendix B. Suppose that the actual network is the optimal system for a budget of B_0. As illustration, the optimal investment of a budget of $B_0 = 10$ is shown in Figure 2, where the width of the arcs represents the capacity.

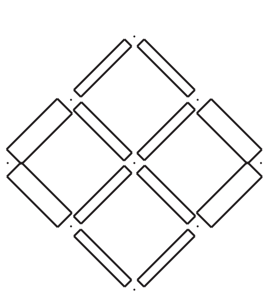

Figure 2. The optimal capacities of the links for a given budget of $B_0 = 10$.

The catastrophic event chooses now an arc j at random decreases its capacity, which was $k_j(c_j)$ to $k_j(c_j - D)$, where D denotes the amount of damage. We assume for simplicity that all other capacities remain unchanged. We assume further that after the catastrophic event the network flows will adjust to the new situation and equal the solution of the max-flow problem under the reduced capacity. Let the expected maximal flow after the event be $U\_(B_0, D)$ (say). Evidently,

$$U\_(B_0, D) \leq U(B_0 - D),$$

because the random shock does not take away some marginal capacity at each arc, but strikes the system much harder by unsystematically hitting fully just one component as illustrated in Figure 3.

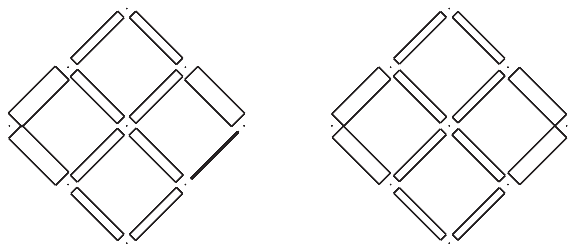

Figure 3. Left: The catastrophic event reduces for instance the capacity of only arc 12. This network can be built with a budget of $B_0 - D$. Right: with the same amount of budget $B_0 - D$ a more effective network could be built as is shown here. The utility of this network is much higher than the one left over by the catastrophe.

The marginal loss in utility caused by the catastrophic event is higher than the marginal increase of utility by a marginal increase of the budget B. Thus considering a basic budget of $B0$ and a return (utility) function $\bar{U}$, which equals $U(B)$ for $B > B_0$ and $U\_(B_0, B_0 - B)$ for $B < B_0$, this function has a kink at B_0 (figure 4).

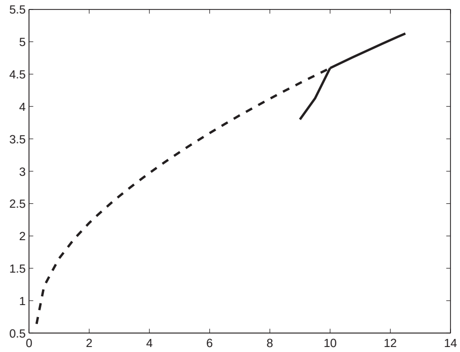

Figure 4. A catastrophe makes the return on investment drop below the historic return curve (The basic stock is B_0 = 10). The function $U(B)$ is shown as a dotted line, the function $\bar{U}(B)$ as a solid line.

In developing countries, where redundant systems are often unaffordable, this effect is magnified because of the threat that a failure anywhere can cause a failure everywhere.

5. Kinked Utility functions for Governments and Risk Aversion

Given the example in section 4 we now prove one of the consequences of such an situation for proposition 1, e.g. that one can not assume that the collective risk premium is small if the marginal return on infrastructure repair after an event is larger than the marginal return on infrastructure investment in a no-event scenario. As a consequence, risk spreading is not an option and therefore the government should behave risk averse in such situations.

Assume that the actual stock value at the beginning of the reference year is B_0 and that a budget of b is foreseen for this year. A possible catastrophic event of random size Z reduces the stocks (w.l.o.g. at the beginning of every year). The stock value after the event and the investment of b is

$$B_0 - Z + b = B_0 + c + X$$

with $c = b - \mathbb{E}(Z)$ is the expected change in stocks and $X = -Z + \mathbb{E}(Z)$ is the zero-mean risk part. We assume that $c > 0$, otherwise the budget would not be sufficient to cover the expected annual damage. The utility function U is assumed to be strictly monotonic and concave. Since we assume that B_0 is large in comparison to b and Z (and therefore w.r.t. c and X), we parameterize the quantities c and X by a small parameter ϵ, i.e. we consider

$$B_0 + \epsilon c + \epsilon X.$$

Introduce the risk premium π as the certainty equivalent, i.e. the solution of

$$U(B_0 + \epsilon c - \pi(\epsilon)) = \mathbb{E}[U(B_0 + \epsilon c + \epsilon X)],$$

that is

$$\pi(\epsilon) = B_0 + \epsilon c - U^{-1}\mathbb{E}[U(B_0 + \epsilon c + \epsilon X)].$$

The concavity of U implies that $\pi(\epsilon) \geq 0$, i.e. the risk premium is positive.

Recall that the Arrow-Lind Theorem states that the collective risk premium $\pi(\epsilon)/\epsilon$ tends to zero with ϵ tending to zero (set $\epsilon = 1/n$ and use proposition 1.) The following proposition shows that this effect disappears for kinked utilities.

Proposition 2. *Assume that* U *is strongly left-and right-sided differentiable at* B_0 *with left side derivative* γ_- *and right side derivative* γ_+, *i.e. there is a* $\delta > 0$ *and a* $K > 0$ *such that for* $v > 0$

$$|U(B_0 + v) - U(B_0) - v\gamma_+| \leq K|v|^{1+\delta}$$

and for $v < 0$

$$|U(B_0 + v) - U(B_0) - v\gamma_-| \leq K|v|^{1+\delta},$$

with $\gamma_+ > 0$. *Assume further that* $\mathbb{E}(|X|^{1+\delta}) < \infty$.

Case A. *If* $\frac{\gamma_- - \gamma_+}{\gamma_-}\mathbb{E}([X + c]^+) > c$, *then*

$$\lim_{\epsilon \downarrow 0} \frac{\pi(\epsilon)}{\epsilon} = \frac{\gamma_- - \gamma_+}{\gamma_-}\mathbb{E}([X + c]^+).$$

Case B. *If* $\frac{\gamma_- - \gamma_+}{\gamma_-}\mathbb{E}([X + c]^+) < c$, *then*

$$\lim_{\epsilon \downarrow 0} \frac{\pi(\epsilon)}{\epsilon} = \frac{\gamma_- - \gamma_+}{\gamma_+}(\mathbb{E}([X + c]^-).$$

Here $[a]^+ = \max(a, 0)$ and $[a]^- = -\min(a, 0)$ are the positive and negative part of a, respectively.

Proof. See Appendix A.

The Arrow-Lind theorem is contained as the special case that U is smooth at a, i.e. $\gamma_+ = \gamma_-$ and that c $=0$. In this case, one sets $\epsilon = 1/n$, i.e. divides the risk among n persons with identical utility and finds that

$$n \cdot \pi n = n \cdot (B_0 - U^{-1}(\mathbb{E}[U(B_0 + X/n))) \to 0.$$

Proposition 2 is valid for small ϵ. In the special case that the utility function is piecewise linear i.e.

$$U(x) = \gamma_+[x - B_0]^+ - \gamma_-[x - B_0]^-,$$

then no limit operation is necessary, since then for every ϵ

$$\pi(\epsilon) = \epsilon\pi(1)$$

with

$$\pi(1) = \begin{cases} \frac{\gamma_- - \gamma_+}{\gamma_-}\mathbb{E}([X + c]^+) & \text{if } \frac{\gamma_- - \gamma_+}{\gamma_-}\mathbb{E}([X + c]^+) > c \quad \text{(Case A)} \\ \frac{\gamma_- - \gamma_+}{\gamma_+}(\mathbb{E}([X + c]^-) & \text{if } \frac{\gamma_- - \gamma_+}{\gamma_-}\mathbb{E}([X + c]^+) < c \quad \text{(Case B).} \end{cases}$$

Notice that in the case of Proposition 2, $\pi(\epsilon)/\epsilon = 0$ only if either $\gamma_- = \gamma_+$ (i.e. there is no kink) or Case B is fulfilled and $\mathbb{E}([X + c]^-) = 0$, which means that $X \geq -c$ or equivalently $Z \leq b$, i.e. that the maximally possible catastrophe is such small that all its damage can be repaired with the annual budget b. We remark that $\mathbb{E}([X + c]^+) = 0$ cannot happen for $c > 0$.

Summarizing, governments should not behave risk neutral when utility functions are kinked after a catastrophe, because marginal utility of reconstruction in the case of an catastrophic (cat-) event can be quite higher than in the case of a no cat-event. This implies that in case of *no* resources for reconstruction a cat-event can have dramatic impacts on the economic performance (as illustrated by the network flow example) while *with* resources the marginal utility of the reconstruction process (e.g. repairing roads) can be very high. In other words, beneath the direct losses the indirect effects can be enormous and therefore have to be included in the risk management decision making process.

6. Discussion

The risk neutrality paradigm for governments with regards to catastrophe risk dates back to the theoretical analysis of Arrow and Lind (1970). This position can and has however been criticized from various perspectives. Risk spreading ability is especially controversial: how many people are needed so that the risk premium for each one approaches zero, what must the wealth of the taxpayers be in this case, what does individual risk aversion have to do with it and is the size of the country important? As discussed, the risk spreading ability can be limited dependent on the answers to the questions above. This paper adds to the discussion the implications of kinked utility functions due to catastrophe events and challenges the risk neutrality paradigm from such an perspective.

Motivated by a network example which ideally represents the infrastructure of a country several rationales that governments have kinked utility functions were given. Basically, whereas the development of infrastructure is a careful, return-maximizing process, natural catastrophes remove capital stock in a less-than optimal way. For example, catastrophes may damage a fraction of a power plant that disables the entire facility, destroy a segment of a road rendering the entire road useless, or attack key telecommunications centers, rather than recent peripheral investments. As a consequence, they can seriously affect the economy as a whole. For example, Honduras was unprepared in terms of resources for rapid recovery when hit by Hurricane Mitch. Many infrastructure reconstruction needs still remain mainly because of insufficient domestic financial resources (Telford et al., 2004). Recognizing, that ”Infrastructure represents, if not the engine, then the ”wheels” of economic activity” (World Development Report, 1994), negative economic effects can be prolonged by a lack of full infrastructure reconstruction. Governments should therefore take these risks into account, at least in their development planning processes.

From a government perspective the analysis has important implications. Risk neutrality can not be assumed and therefore risk management strategies should be considered. Generally speaking, risk instruments to lessen the risk of large contingent liabilities for the government in the case of a cat event, can be separated into ex-post resources or pro-active measures. While the former includes taxation, budget diversion and outside assistance, the latter includes insurance, reserve funds, structural mitigation measures and redundant systems. A discussion of all these measures is not possible here (see Mechler, 2004b; Hochrainer, 2006) and we restrict ourselves on a discussion of possible pro-active measures for the network example in section 4.

Mitigation measures like strengthening bridges in key areas (important network knots) would decrease the marginal utility loss in a cat-event scenario. If mitigation was enough to withstand a natural disaster with a certain impact, this cat event would have no further consequences. If mitigation lessens the impact, the marginal utility loss in the cat-event would be *less* than the marginal utility loss in the cat-event with no mitigation, but on the other hand, the absolute utility in the cat event with mitigation measures would be *higher* than in the scenario without mitigation. Hence, mitigation measures are efficient if the marginal investment utility is high. Redundant systems are similar to (physical) mitigation measures, but rather than strengthening constructions to lessen the impact of a disaster, redundant systems could reduce the risk of total failure if they are sufficiently spatially uncorrelated with the risk, e.g. earthquakes are usually localized events and therefore redundant systems have to be separated from such areas (it should be noted that only a single event is assumed here, further research in the future will take into account multi-event scenarios). In this case, to develop an effective and cheap redundant system, risk based infrastructure interdependencies should be assessed and network improvement to counteract disastrous events analyzed (Brown et al., 2004; Poorzahedy and Bushehri, 2005).

In contrast to structural mitigation measures pro-active financial risk management instruments like insurance, catastrophe bonds or reserve fund arrangements do not lessen the physical impact of disasters, but ensure that money is available without delay after the cat-event so that a quick repair and return to the status quo is possible. Using the network example above, financial instruments are preferable to mitigation measures when marginal utility without a cat event is low, but absolute utility is high.

Further research includes the analysis of investment strategies in risk management instruments for different marginal return rates as well as the incorporation of simultaneous multi event scenarios in network systems.

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Appendix A: Proof of Proposition 2

Set $Y = X + c$. Notice that $\mathrm{E}(|X|^{1+\delta}) < \infty$ implies that also $\mathrm{E}(|X|^{1+\delta}) < \infty$. Recall that the risk premium $\pi(\mathbb{E})$ is defined by the relation

$$U(B_0 + \epsilon c - \pi(\epsilon)) = \mathbb{E}[U(B_0 + \epsilon Y)] \tag{2}$$

i.e.

$$\pi(\epsilon) = B_0 + \epsilon c - U^{-1}(\mathbb{E}[U(B_0 + \epsilon Y)]). \tag{3}$$

We show first that there is a constant K_1 such that

$$\pi(\epsilon) \leq K_1\epsilon. \tag{4}$$

First, the definition implies that $\pi(\epsilon)$ tends to 0 as ϵ tend to 0. Since U is strictly increasing and concave in a neighborhood of B_0, its inverse U^{-1} must satisfy

(5) $$|U^{-1}(y) - B_0| \leq K_0 |y - U(B_0)|$$

for some $K_0 > 0$ and for y in some neighborhood of $U(B_0)$.

The strong differentiability assumption implies that

$$|U(B_0 + \epsilon Y) - U(B_0)| \leq \epsilon \cdot \max(\gamma_+, \gamma_-)|Y| + \epsilon^{1+\delta} \cdot K|Y|^{1+\delta},$$

i.e. by taking expectations and for $\epsilon \leq 1$,

$$|\mathbb{E}[U(B_0 + \epsilon Y)] - U(B_0)| \leq C \cdot \epsilon,$$

where $C = \max(\gamma_+, \gamma_-)\mathbb{E}|Y| + K\mathbb{E}|Y|^{1+\delta}$. Setting $y = \mathbb{E}[U(B_0 + \epsilon Y)]$ in (5) one obtains for sufficiently small ϵ

$$|U^{-1}(\mathbb{E}[(B_0 + \epsilon Y)]) - B_0| \leq K_0 |\mathbb{E}[U(B_0 + \epsilon Y)] - U(B_0)| \leq CK_0\epsilon$$

and substituting $\epsilon c - \pi(\epsilon)$ for $U^{-1}(\mathbb{E}[(B_0 + \epsilon Y)]) - B_0$ one gets

(6) $$|\epsilon c - \pi(\epsilon)| \leq CK\epsilon$$

which implies (4) with $K_1 = CK_0 + c$.

Now we will approximate both sides of (2). First, the right hand side of (2) is approximated. From the strong differentiability one gets with $[Y]^+ = \max(Y, 0)$ and

$[Y]^- = -\min(Y, 0) = [Y]^+ - Y$

$$|U(B_0 + \epsilon Y) - U(B_0) - \gamma_+\epsilon\mathbb{E}[Y]^+ + \gamma_-\epsilon[Y]^-| \leq K\epsilon^{1+\delta}|Y|^{1+\delta}$$

and therefore, by taking the expectation on both sides, one gets further

(7) $$|\mathbb{E}[U(B_0 + \epsilon Y)] - U(B_0) - \gamma + \epsilon\mathbb{E}[Y]^+ + \gamma - \epsilon\mathbb{E}[Y]^-| \leq K\epsilon^{1+\delta}\mathbb{E}[|Y|^{1+\delta}].$$

For the approximation of the left side of (3) one has to distinguish Case A and Case B.

Case A. If $\epsilon c < \pi(\epsilon)$, then

(8) $$|\mathbb{E}[U(B_0 + \epsilon c - \pi(\epsilon))] - U(B_0) - \gamma - (\epsilon c - \pi(\epsilon))| \leq K[\epsilon c + \pi(\epsilon)]^{1+\delta} \leq \epsilon^{1+\delta}K_2,$$

where $K_2 = K(c + K_1)^{1+\epsilon}$. Putting (7) and (8) together, using (6) and the identity (3) one obtains that

$$|\gamma + \epsilon\mathbb{E}[Y]^+ - \gamma_-\epsilon\mathbb{E}[Y]^- + \gamma_-(\pi(\epsilon) - \epsilon c)| \leq \epsilon^{1+\delta}K\mathbb{E}[|Y|^{1+\delta}] + \epsilon^{1+\delta}K_2.$$

Therefore, dividing by ϵ and letting ϵ go to zero, one obtains

(9) $$\gamma_+\mathbb{E}([Y]^+) - \gamma_-\mathbb{E}([Y]^-) + \gamma_- \frac{\pi(\epsilon) - \epsilon c}{\epsilon} \to 0.$$

Notice that $c = \mathbb{E}(Y) = \mathbb{E}[Y]^+ - \mathbb{E}[Y]^-$. Replacing $\mathbb{E}[Y]^-$ in (9) by $\mathbb{E}[Y]^+ - c$ one gets finally in Case A

$$\frac{\pi(\epsilon)}{\epsilon} \to \frac{\gamma_- - \gamma_+}{\gamma_-}\mathbb{E}[Y]^+.$$

Notice that in turn, if Case A is fulfilled, then $\epsilon c < \pi(\epsilon)$ eventually.

Case B. Assume now that $\epsilon c > \pi(\epsilon)$. Then

(10) $$|\mathbb{E}[U(B_0 + \epsilon c - \pi(\epsilon))] - U(B_0) - \gamma_+(\epsilon c - \pi(\epsilon))| \leq K[\epsilon c + \pi(\epsilon)]^{1+\delta} \leq \epsilon^{1+\delta}K_2.$$

Putting (7) and (10) together, using (6) and the identity (3) one obtains that

$$|\gamma_+\epsilon\mathbb{E}[Y]^+ - \gamma_-\epsilon\mathbb{E}[Y]^- + \gamma_+(\pi(\epsilon) - \epsilon c)| \leq \epsilon^{1+\delta}K\mathbb{E}[|Y|^{1+\delta}] + \epsilon^{1+\delta}K_1.$$

Therefore, dividing by ϵ and letting ϵ go to zero, one obtains

(11) $$\gamma_+\mathbb{E}([Y]^+) - \gamma_-\mathbb{E}([Y]^-) + \gamma_+\frac{\pi(\epsilon) - \epsilon c}{\epsilon} \to 0.$$

Replacing $\mathbb{E}[Y]^+$ in (11) by $\mathbb{E}[Y]^- + c$ on gets finally in Case B

$$\frac{\pi(\epsilon)}{\epsilon} \to \frac{\gamma_- - \gamma_+}{\gamma_+}\mathbb{E}[Y]^-.$$

Again, in turn, in Case B eventually $\epsilon c > \pi(\epsilon)$.

Appendix B: Specification of the Network example

The costs c_i as functions of the capacities k_i for the example shown in Figure 1 were specified as

$$c_i(k_i) = \alpha_i k_i + \beta_i k_i^2,$$

for the twelve arcs, $i = 1, \ldots, 12$, where the constants α and β were chosen as

α = [0.1, 0.09, 0.12, 0.11, 0.13, 0.08, 0.1, 0.1, 0.09, 0.11, 0.12, 0.11]

β = [0.3, 0.31, 0.27, 0.26, 0.26, 0.33, 0.31, 0.29, 0.3, 0.3, 0.31, 0.28]

Thus the marginal costs increase with increasing capacity. The capacities as functions of the costs are

$$k_i(c_i) = \sqrt{\frac{\alpha^2}{4\beta^2} + \frac{c_i}{\beta}} - \frac{\alpha}{2\beta}.$$

For every fixed budget B, the optimal investment into the arcs was calculated. Denote by f_i the flow on arc i. The max-flow problem under budget constraints is:

Maximize $F := f_1 + f_2$
under the constraints that
$f_1 = f_3 + f_4$
$f_2 = f_5 + f_6$
$f_3 = f_7$
$f_4 + f_5 = f_8 + f_9$
$f_6 = f_{10}$
$f_7 + f_8 = f_{11}$
$f_9 + f_{10} = f_{12}$
$f_i \le k_i \quad i = 1,..., 12$
$\alpha_i k_i + \beta_i k_i^2 = c_i \quad i = 1,..., 12$
$\sum_{i=1}^{12} c_i \le B$

The decision variables are the flows f_i and the provided capacities k_i. For every budget B in the range $1 \le B \le 13$, the optimal value $U(B)$ of the maximization problem was calculated. Figure 4 shows the function $B \mapsto U(B)$ as a dotted line. It was then assumed that the optimal network for budget $B_0 = 10$ is affected in such a way that an arc is chosen at random with equal probability 1/12 and its capacity is dropped form k_i to k_i' in such a way that $\alpha_i k_i' + \beta_i k_i'^2 = \alpha_i k_i + \beta_i k_i^2 - D$. The expected maximal flow for the partly destroyed network as a function of the damage D is depicted as a solid line in Figure 4.